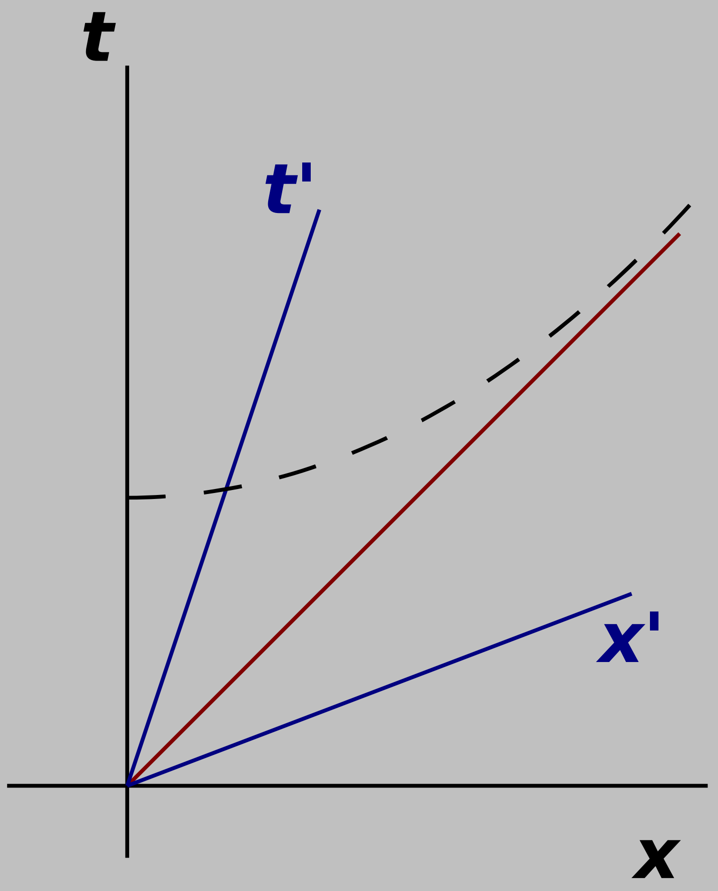
Understanding Relativity With Spacetime Diagrams

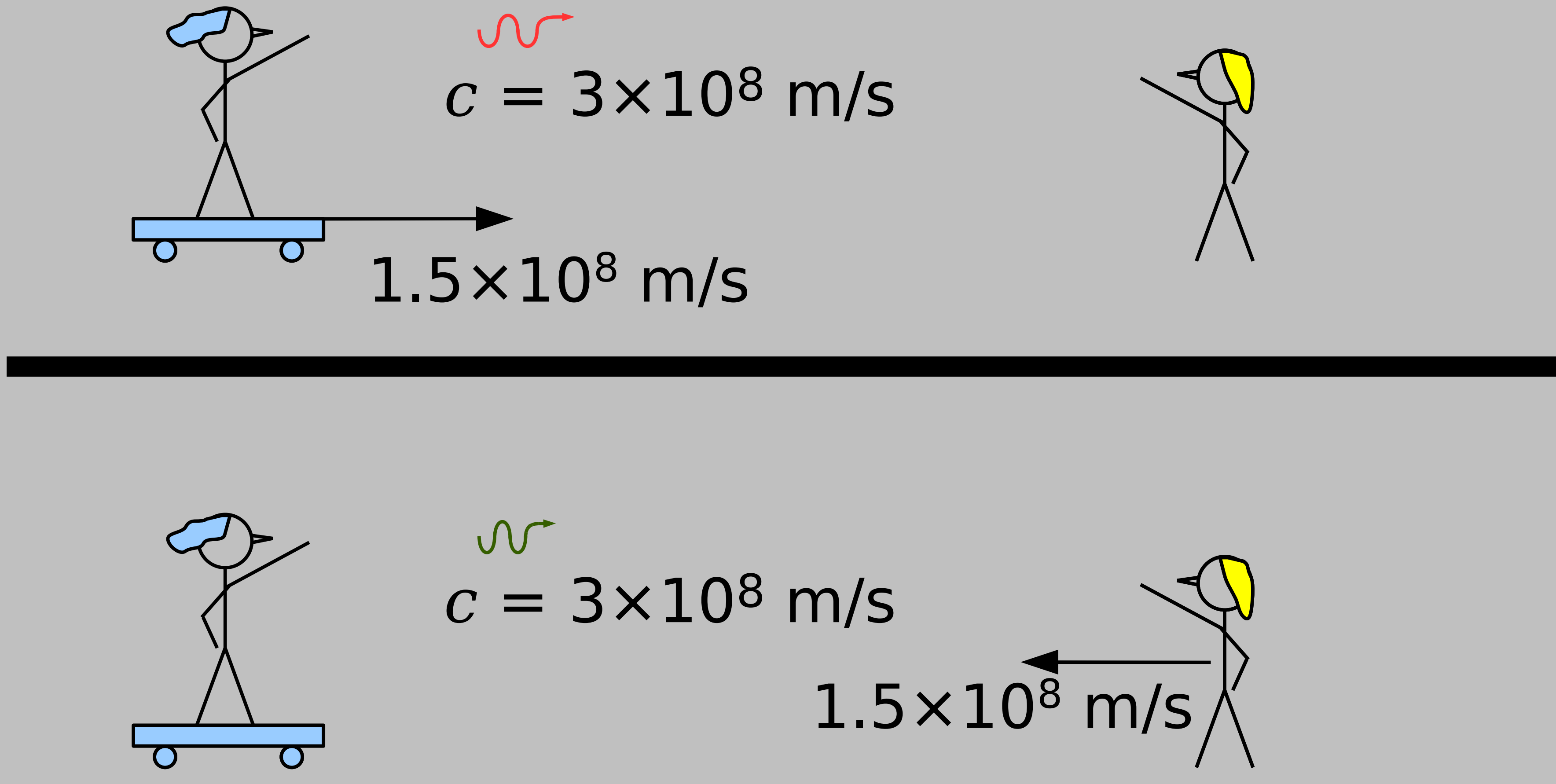


Dr. Rob Knop

MICA (www.mica-vw.org)

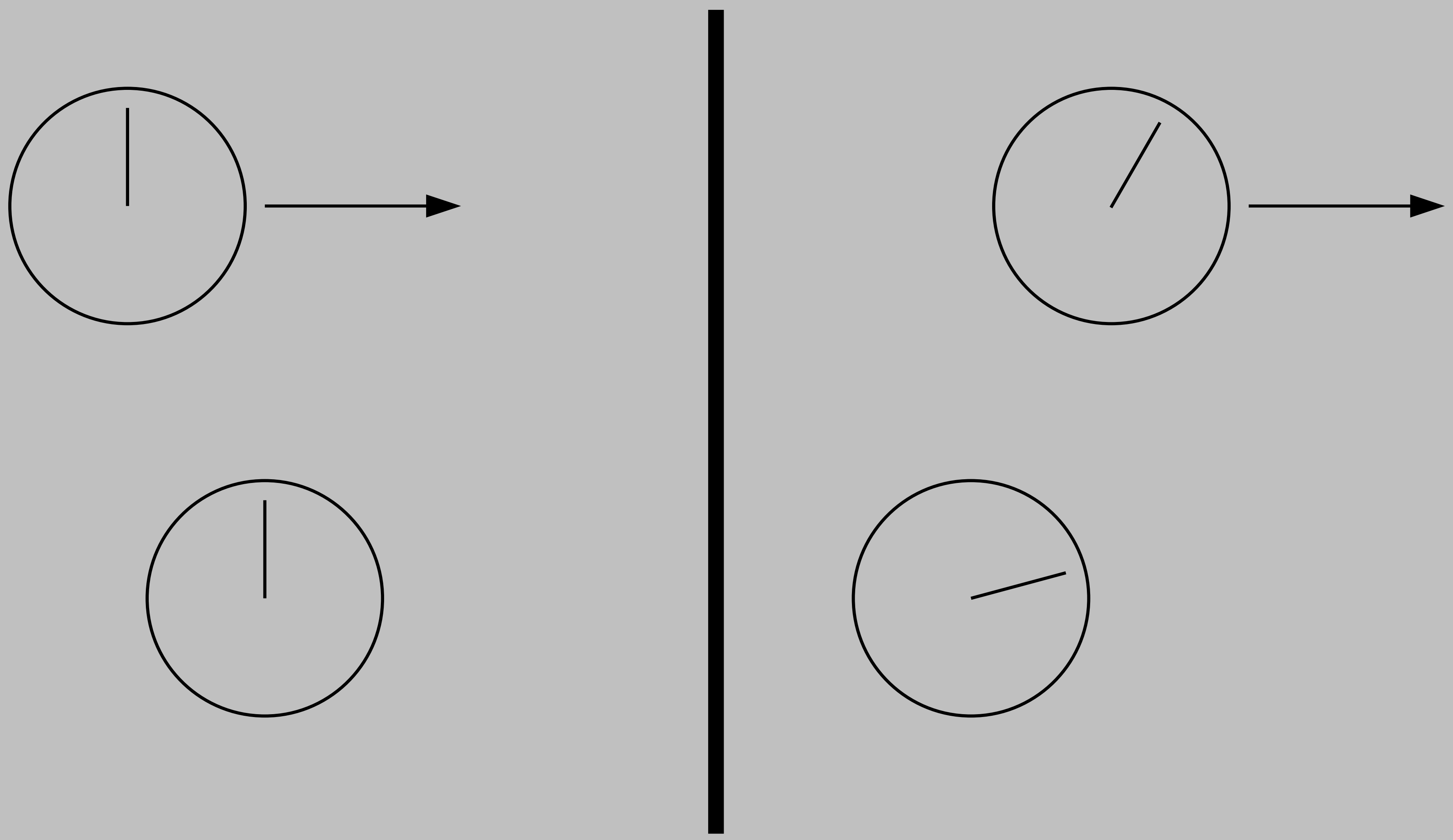
Second Life, 2009-04-10

Special Relativity : the speed of light in a vacuum is the same as measured by any observer.



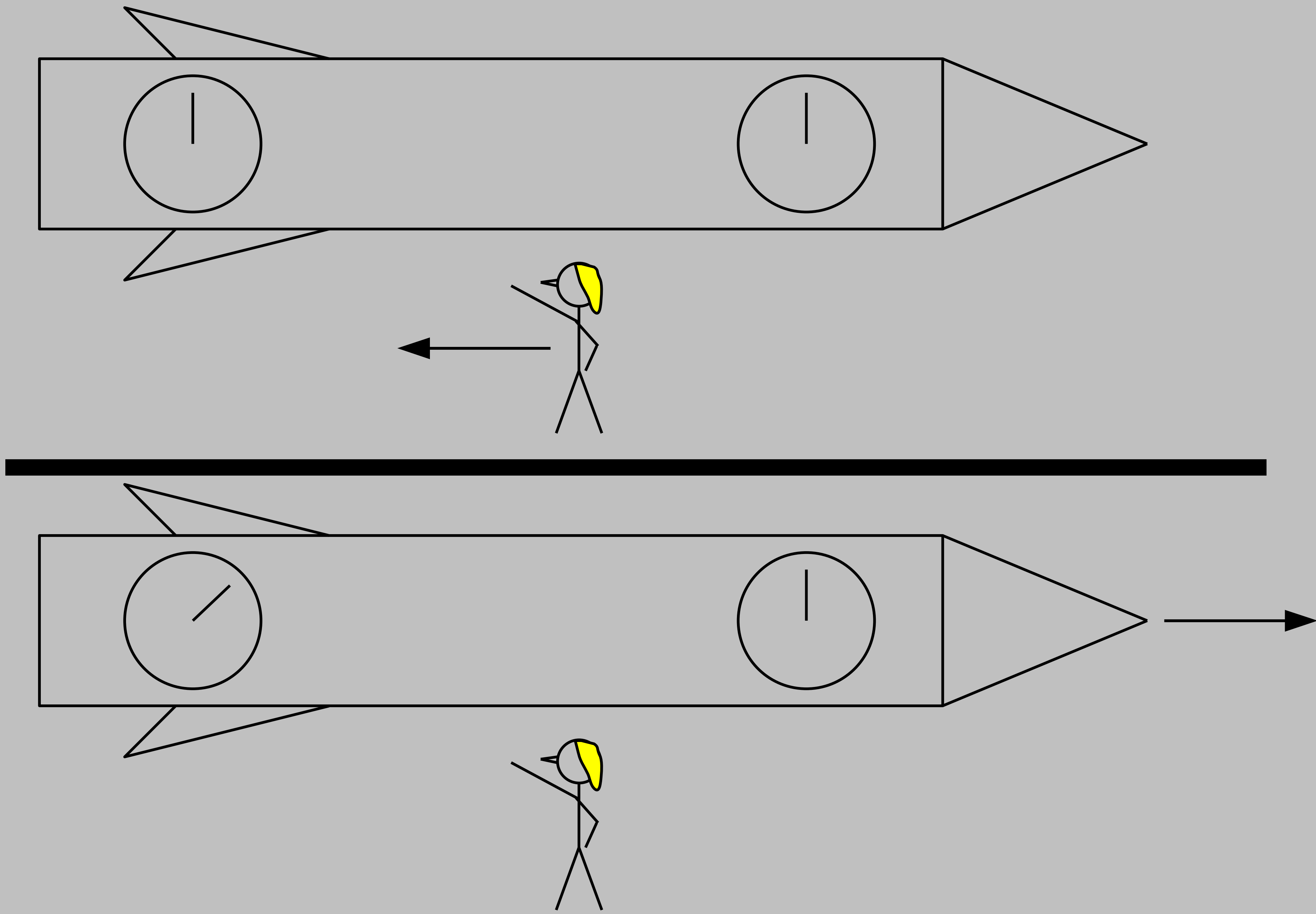
It's not intuitive, but it's true... *and it has consequences.*

SR Effect #1 : Time Dilation



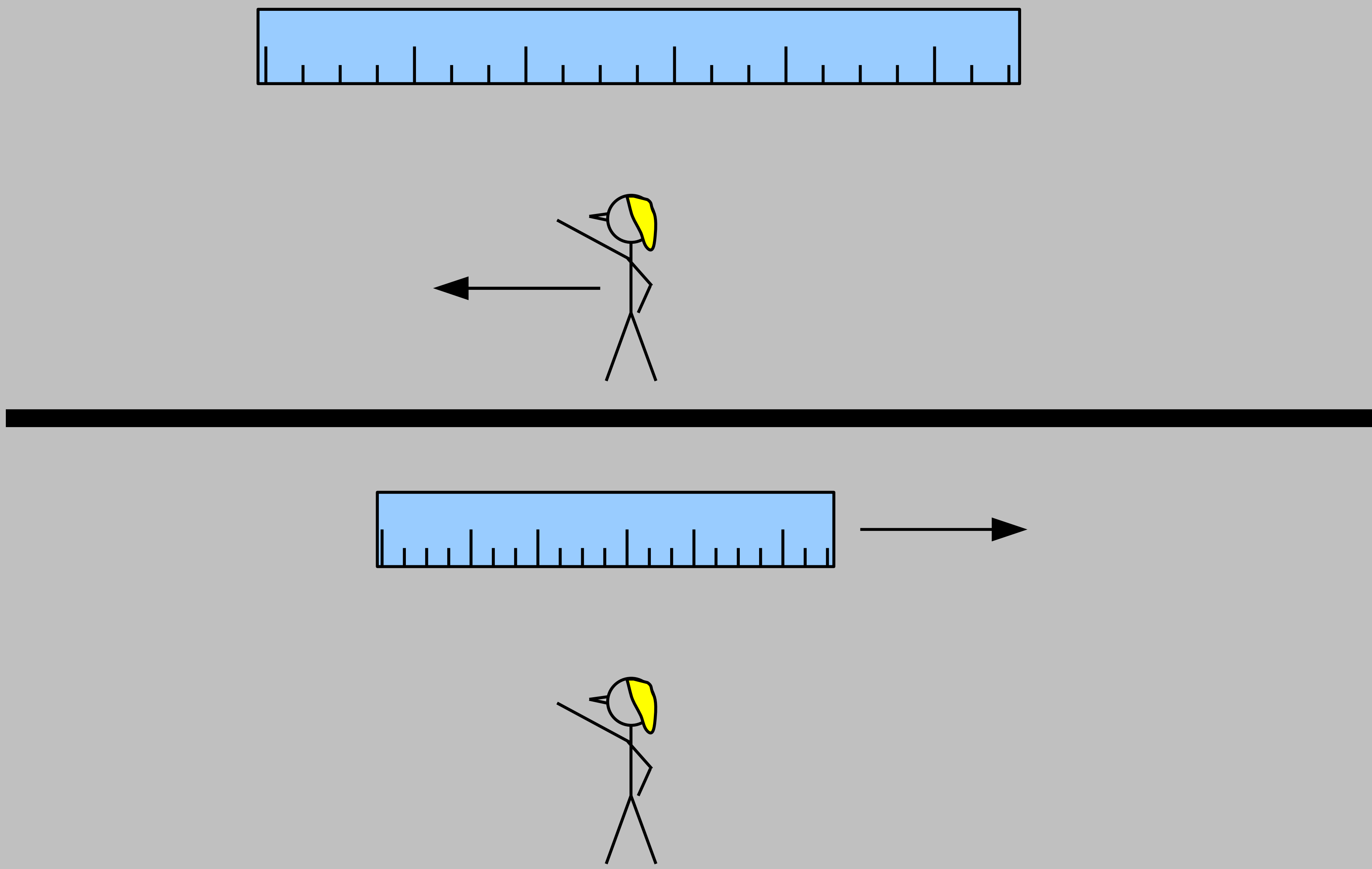
Moving clocks run slow.

SR Effect #2 : Simultaneity



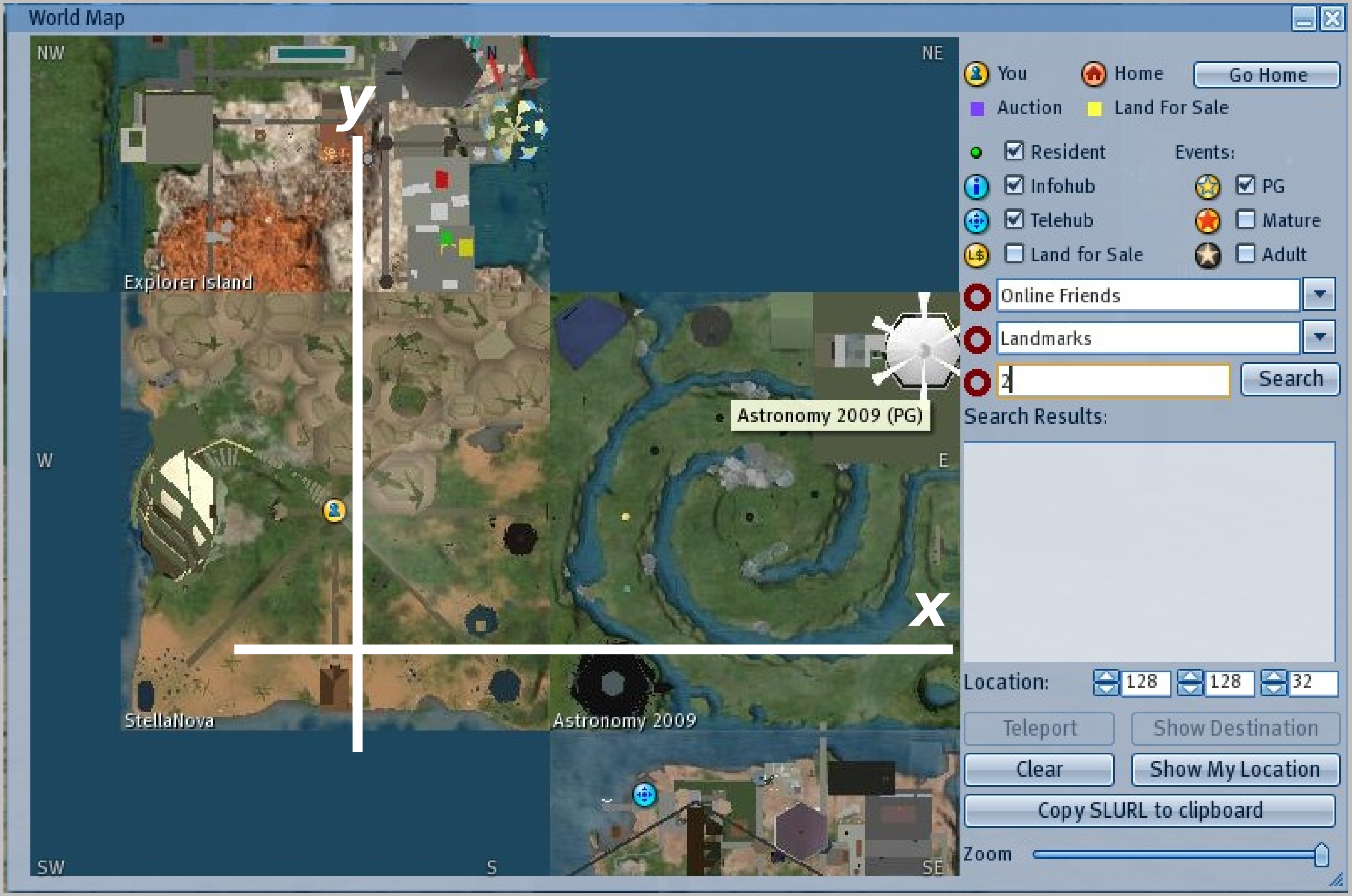
Clocks synchronized in one frame aren't in another.

SR Effect #3 : Lorentz Contraction

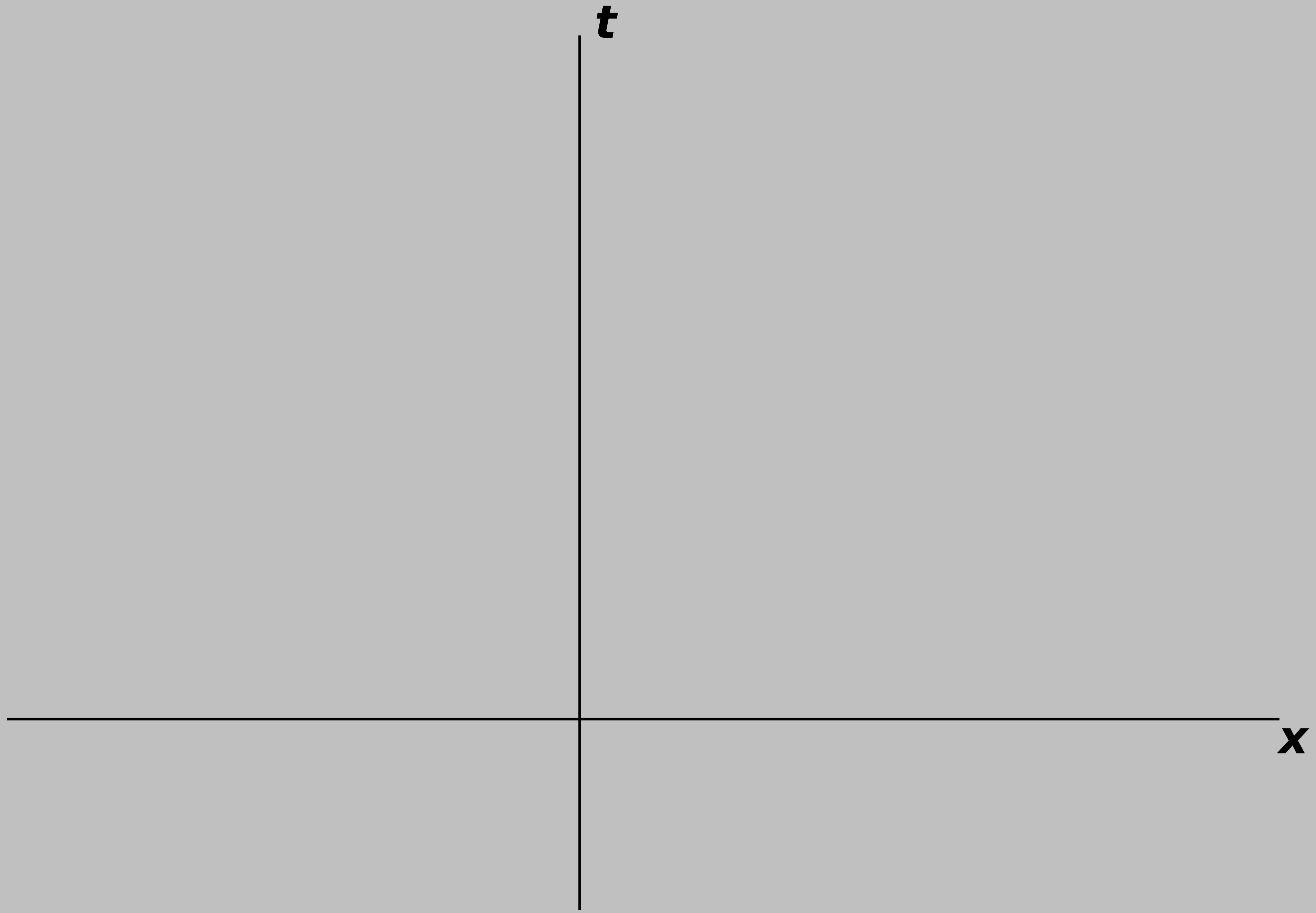


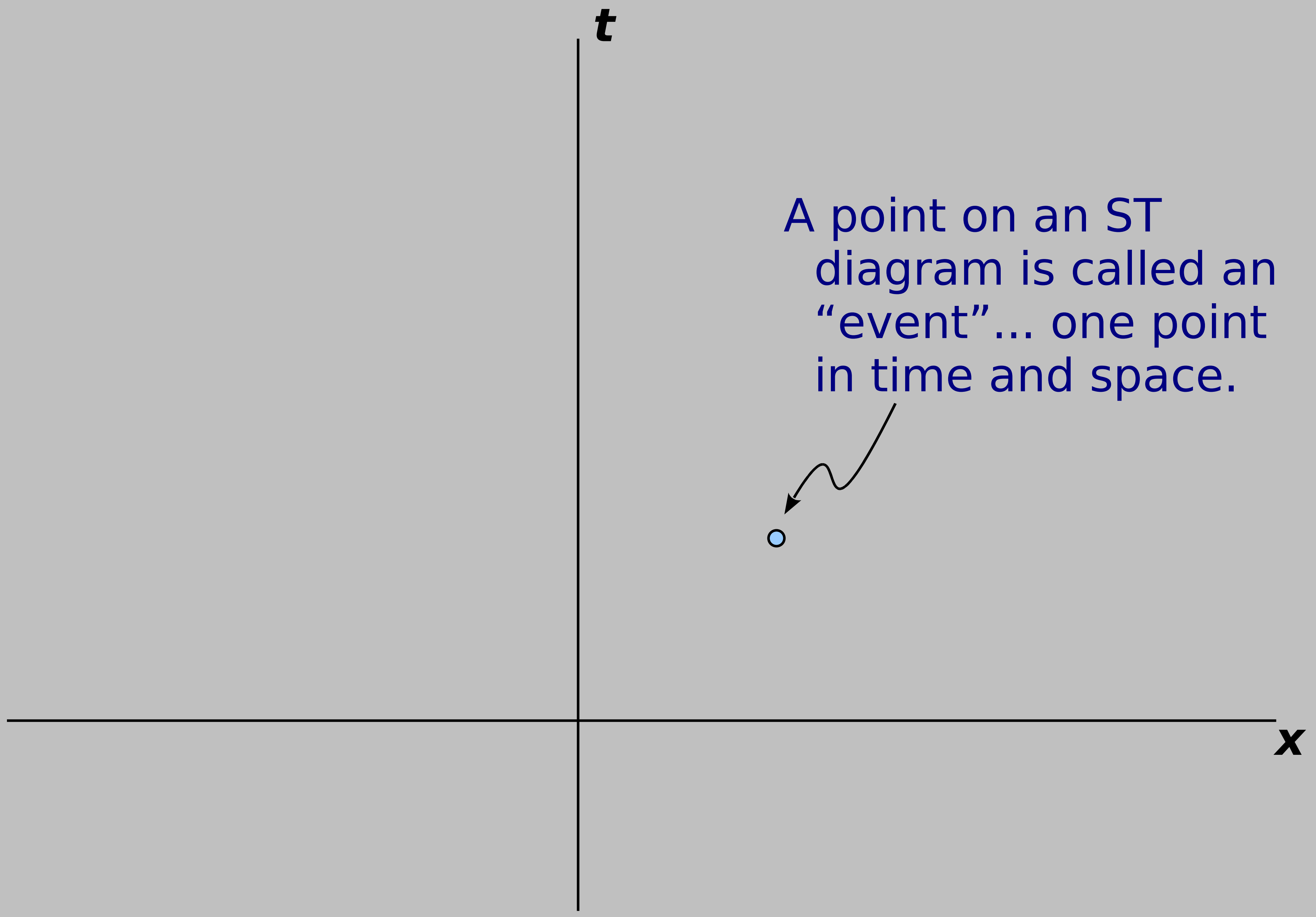
Moving objects *are* shorter along the direction of motion.

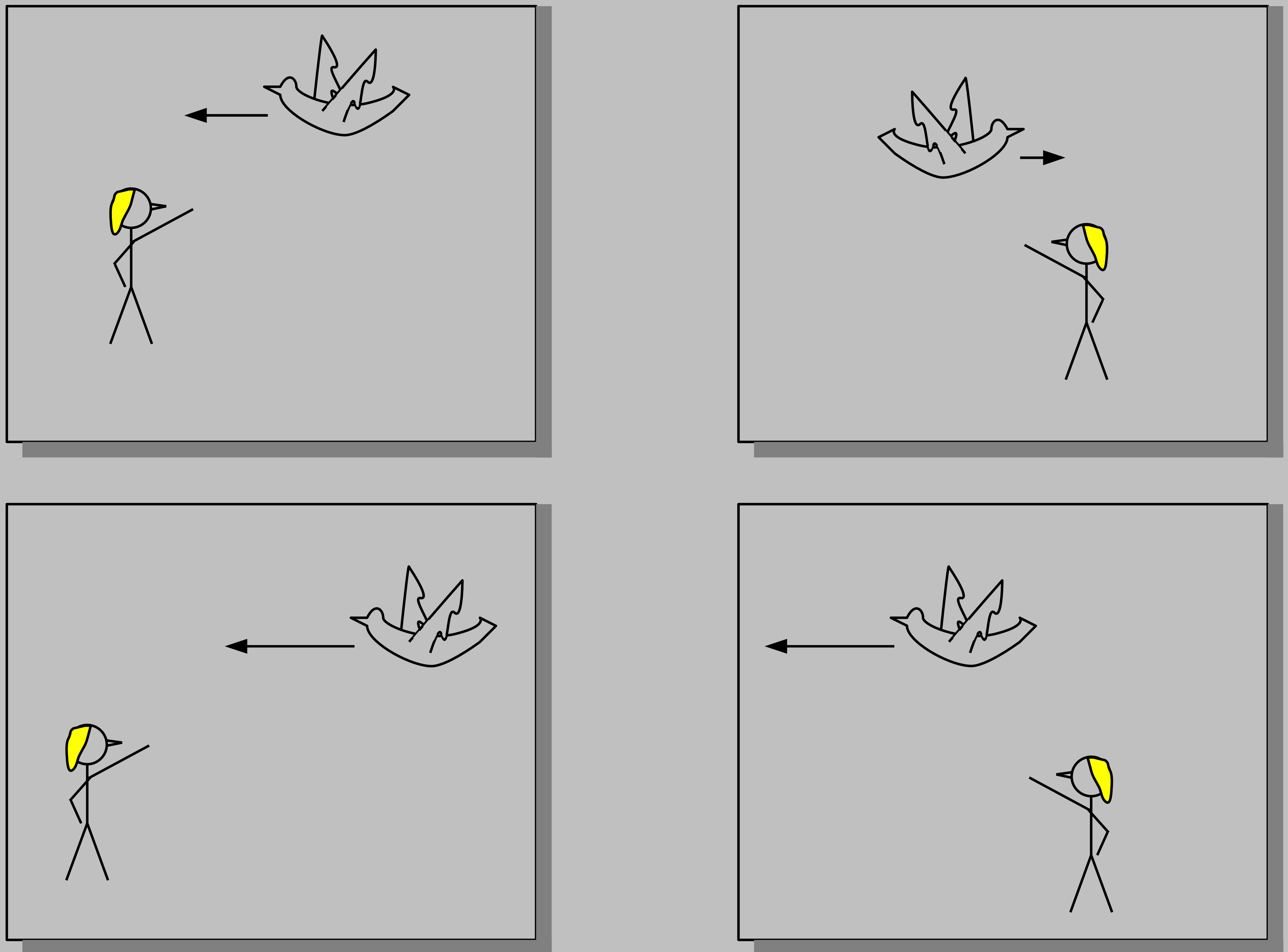
Space – Space Diagrams



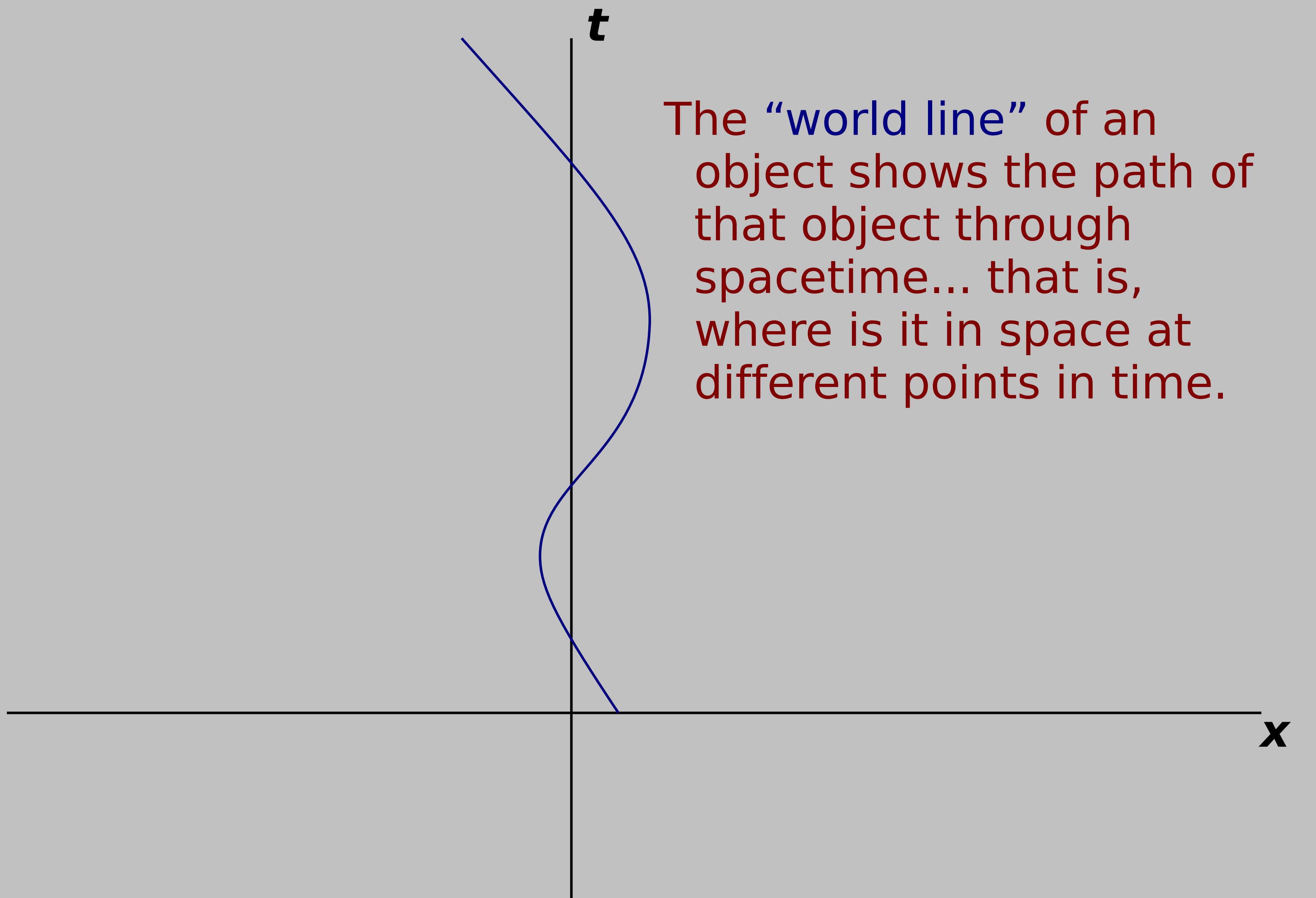
Space – Time Diagrams



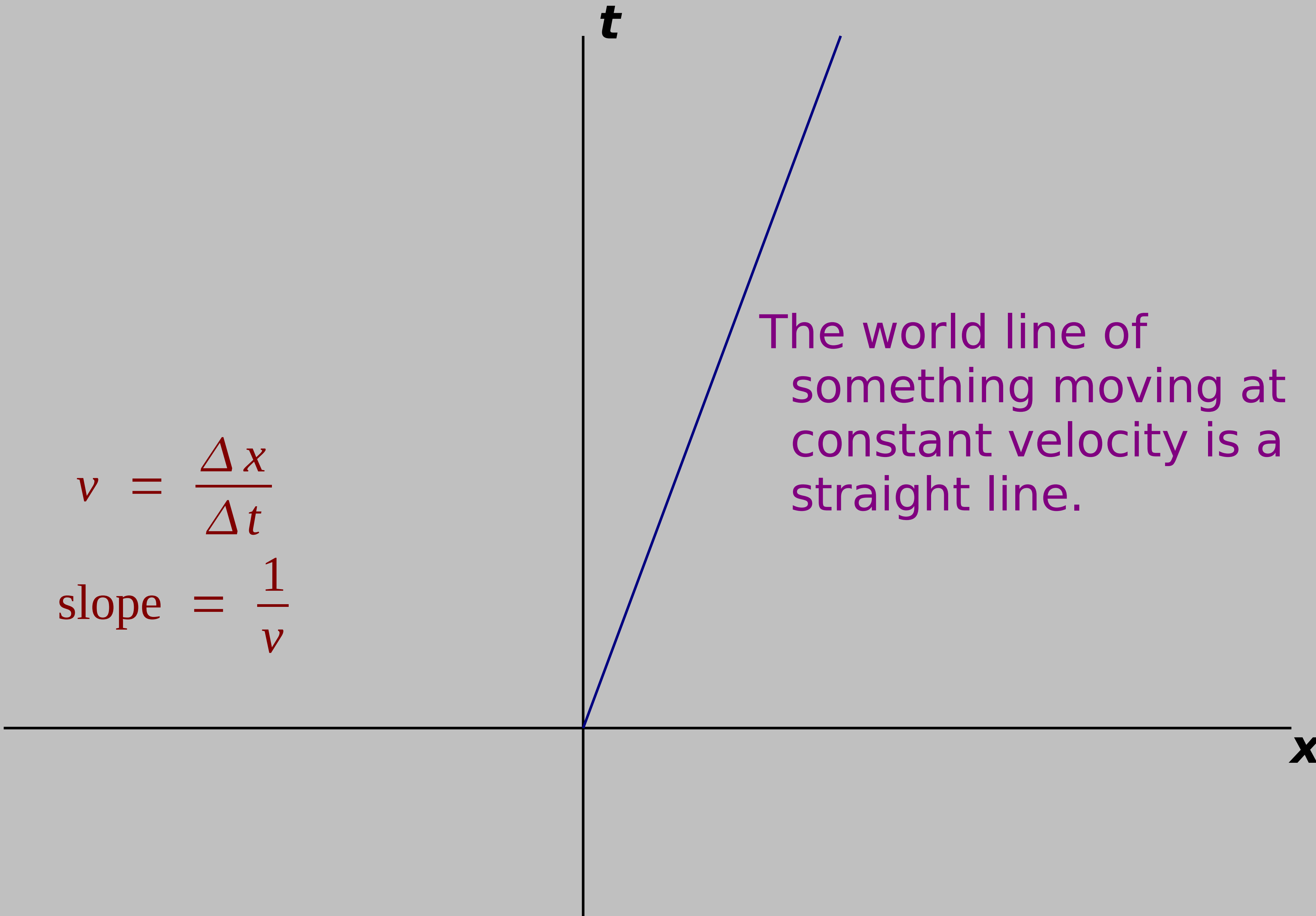

t
x
A point on an ST diagram is called an "event"... one point in time and space.



World Lines



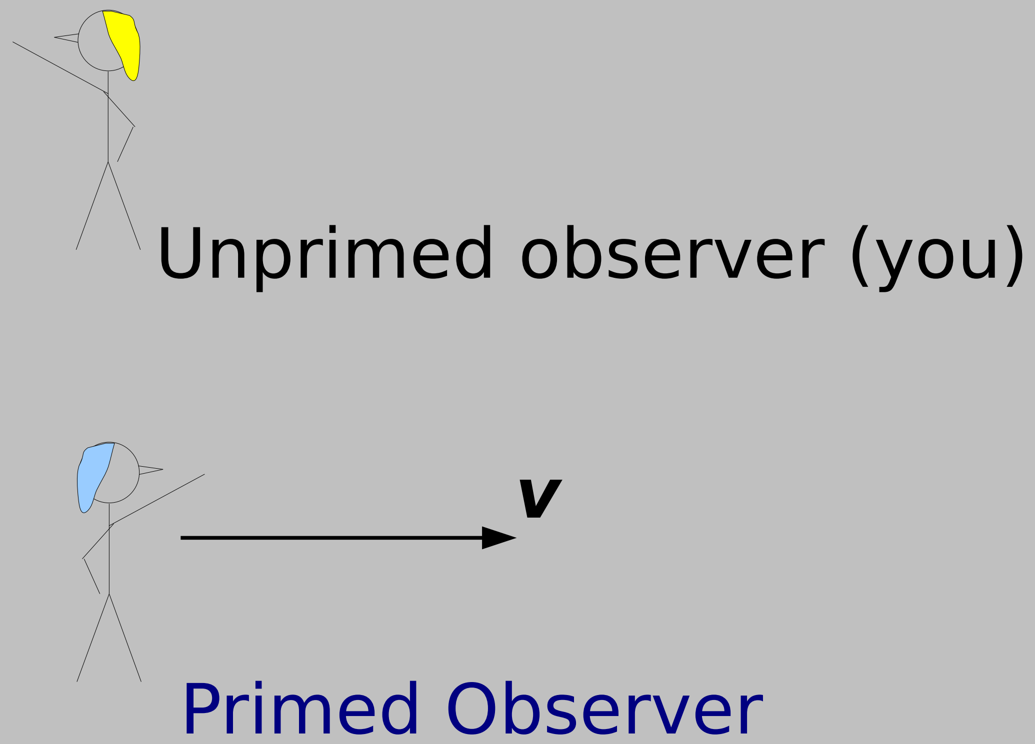
The "world line" of an object shows the path of that object through spacetime… that is, where is it in space at different points in time.

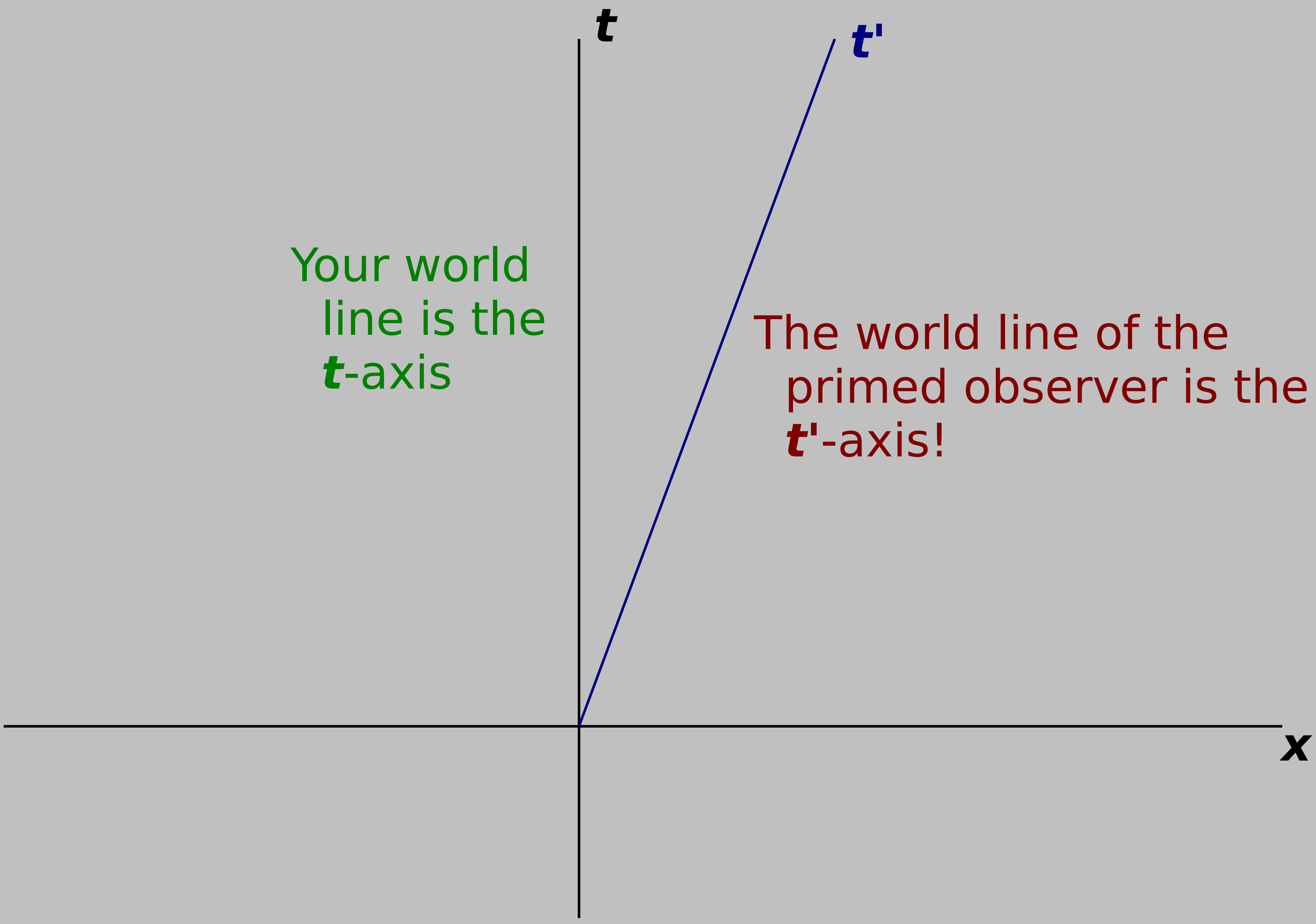


The world line of something moving at constant velocity is a straight line.

$$v = \frac{\Delta x}{\Delta t}$$

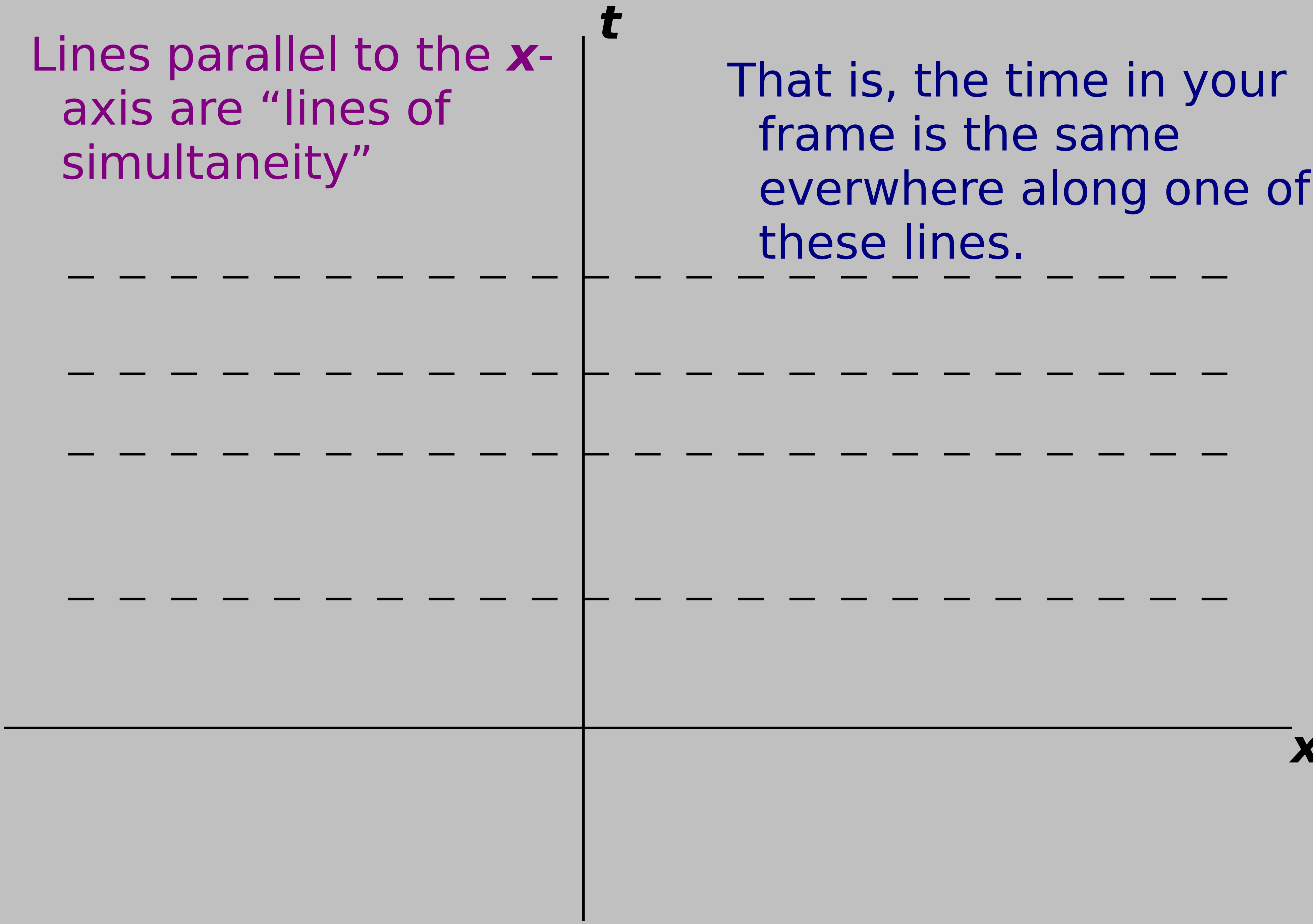
$$\text{slope} = \frac{1}{v}$$

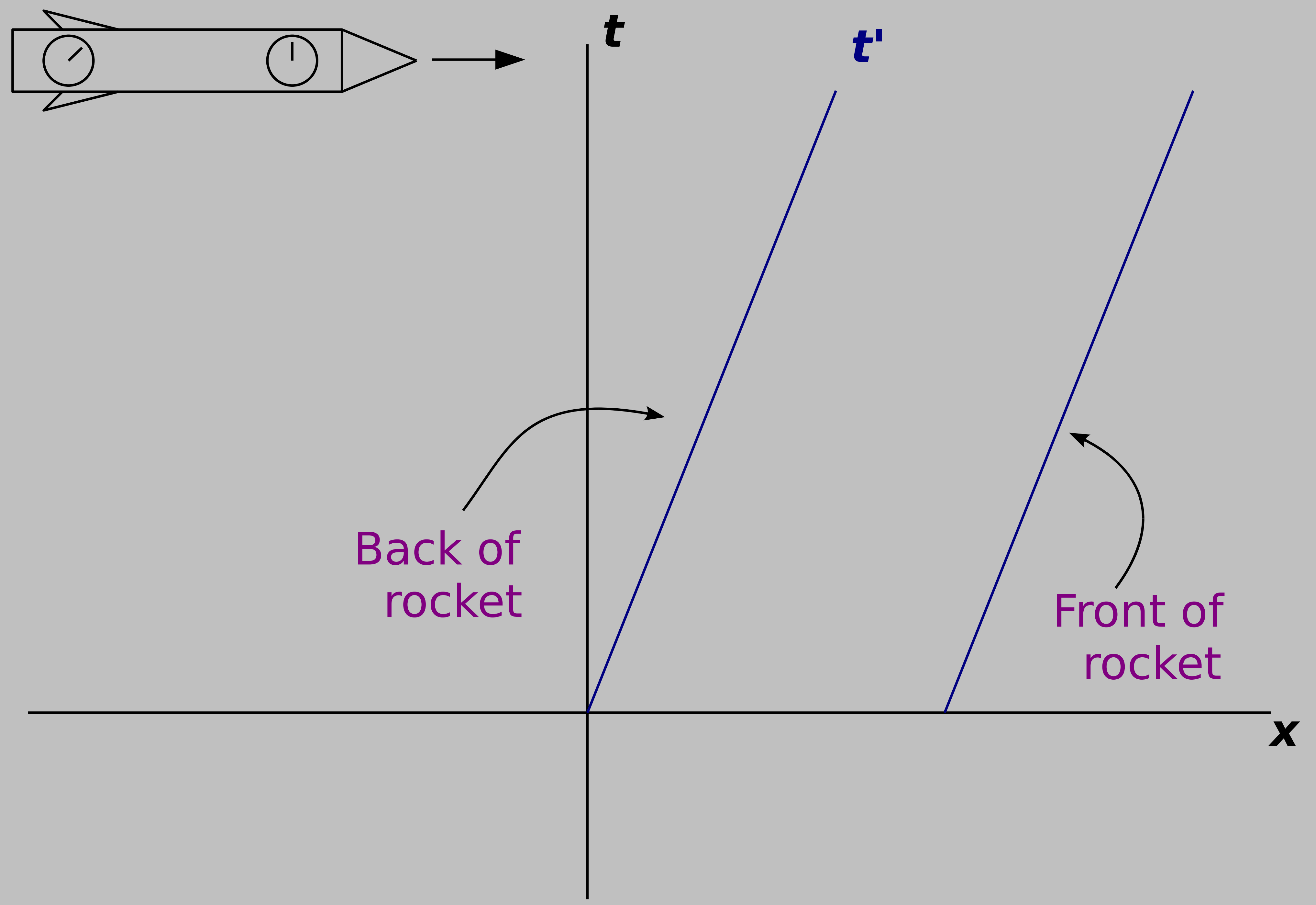

Unprimed observer (you)
v
Primed Observer


t
t'
Your world line is the t-axis
The world line of the primed observer is the t'-axis!
x

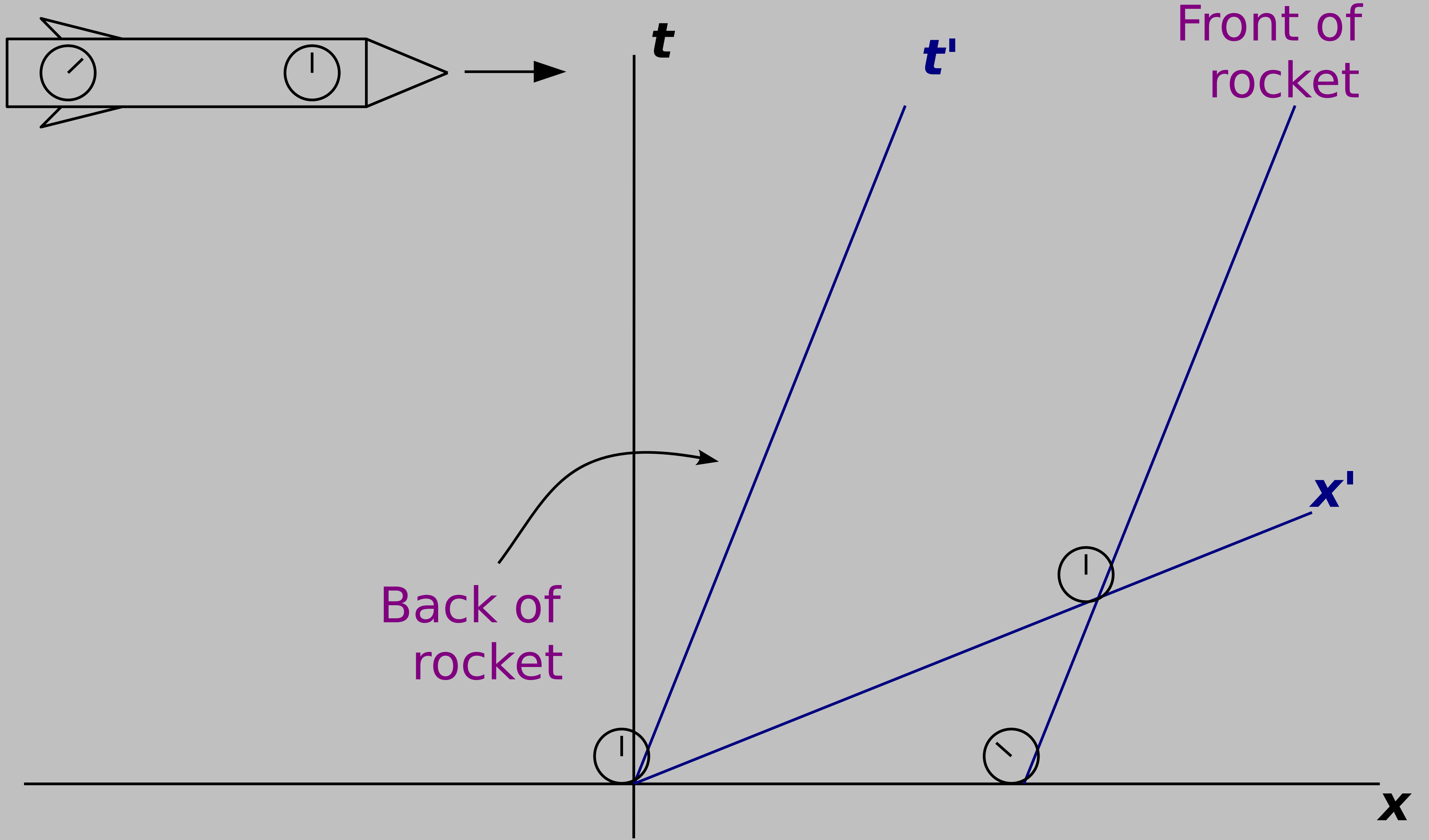
Lines parallel to the ***x***-axis are “lines of simultaneity”

That is, the time in your frame is the same everwhere along one of these lines.

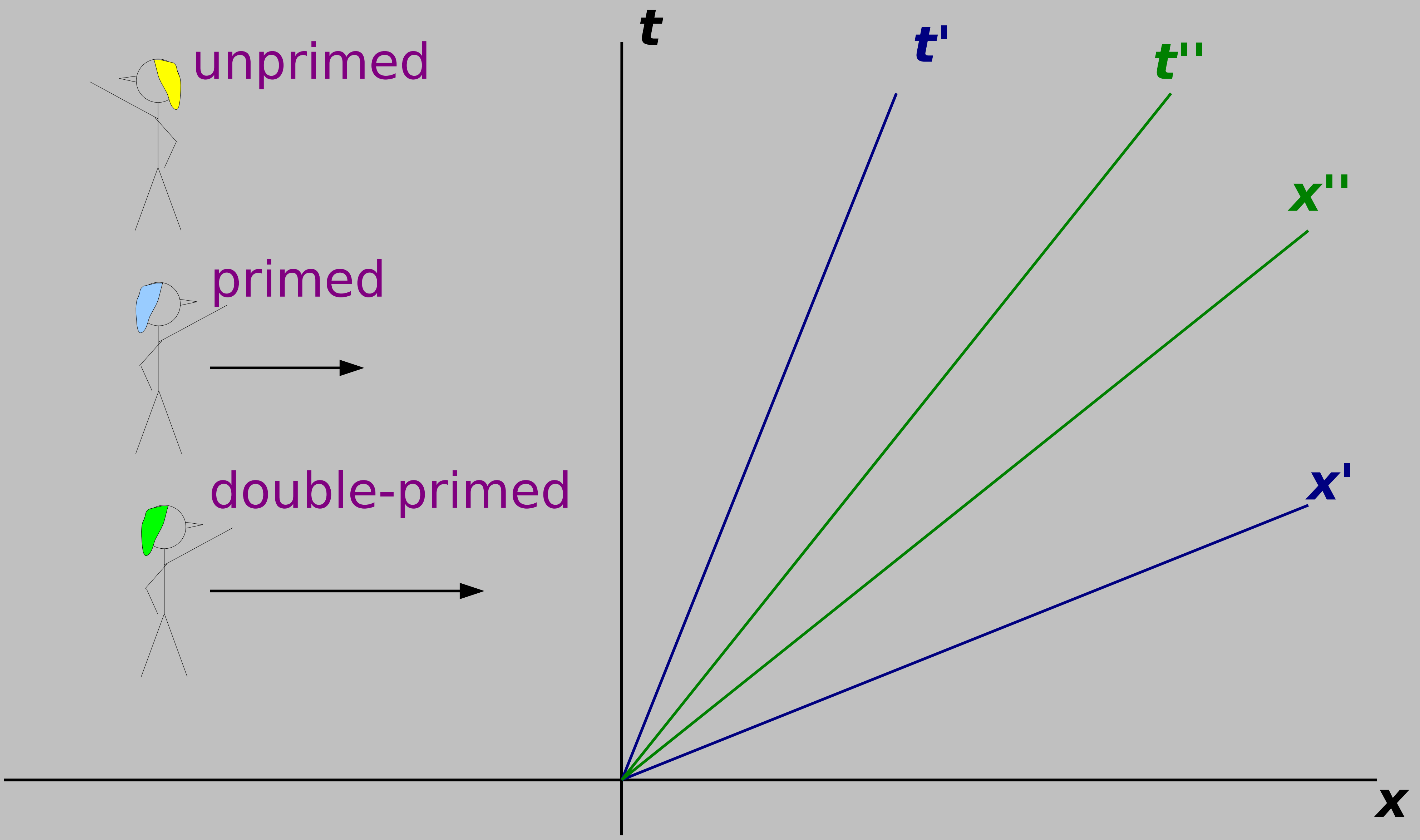




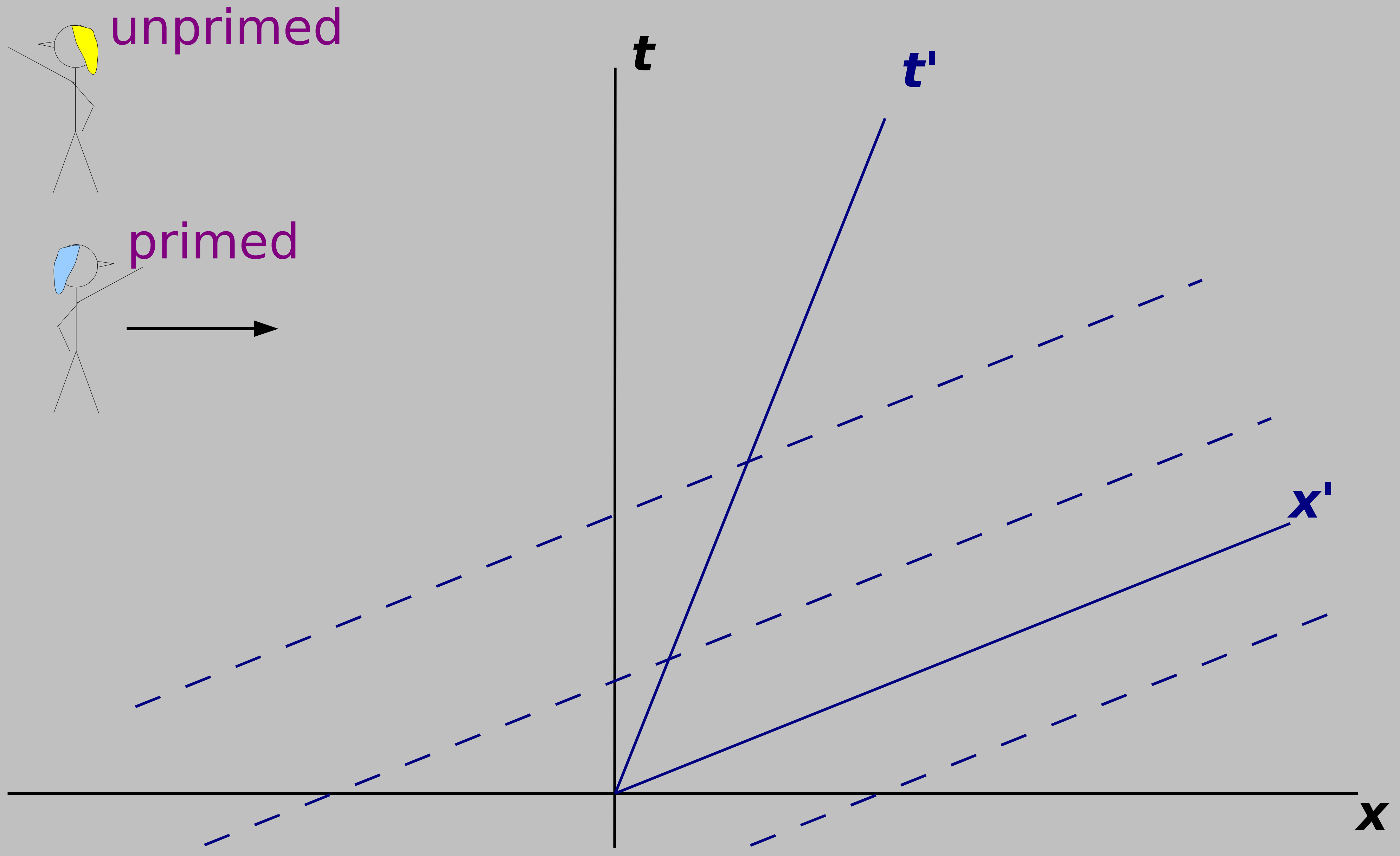
t
t'
Back of rocket
Front of rocket
x



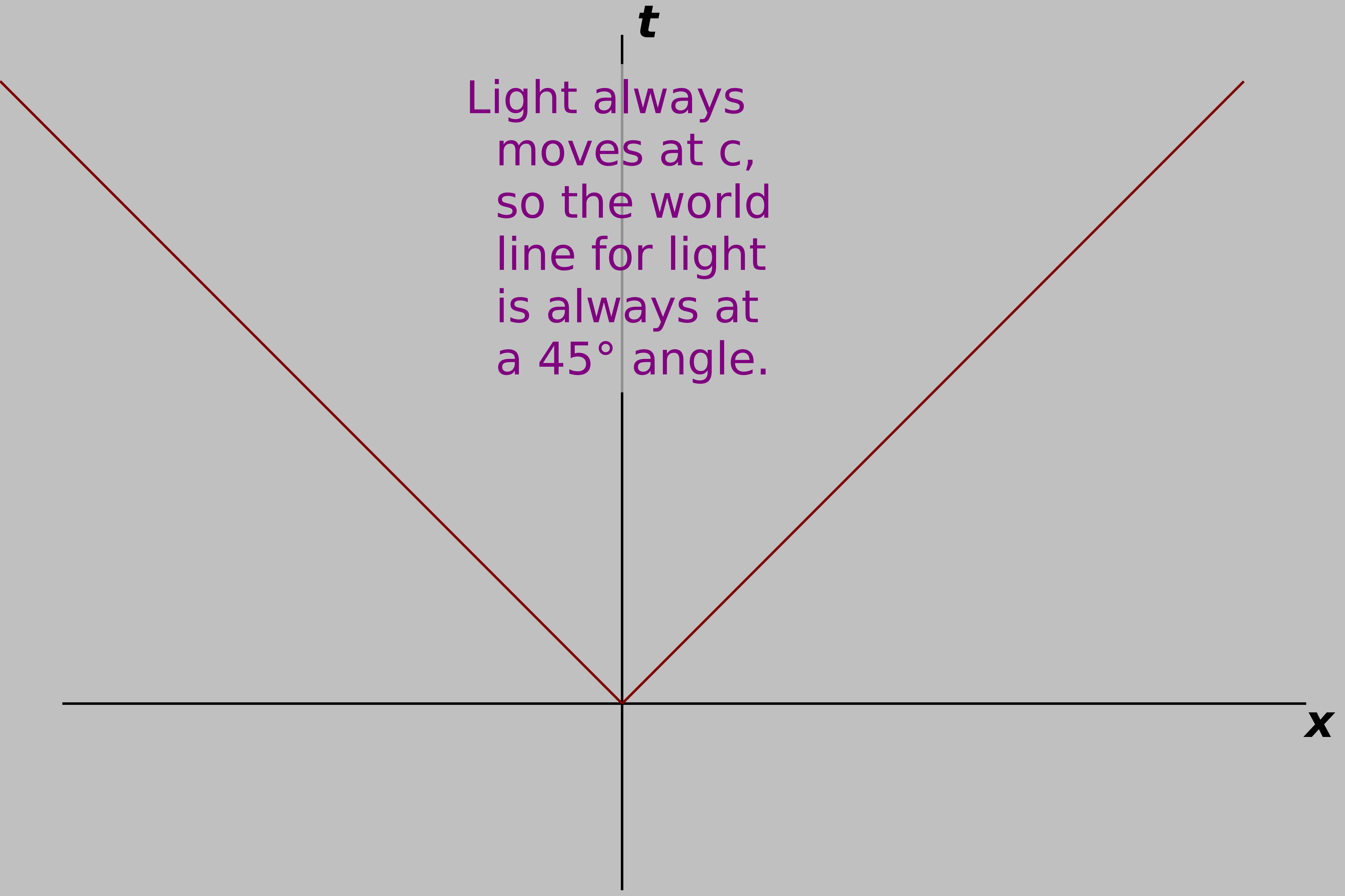
In our frame, when the back clock is at t=0, the front clock is at ***t***<0. The front clock gets to ***t***=0 *later*. That tells us the orientation of the ***x'***-axis.



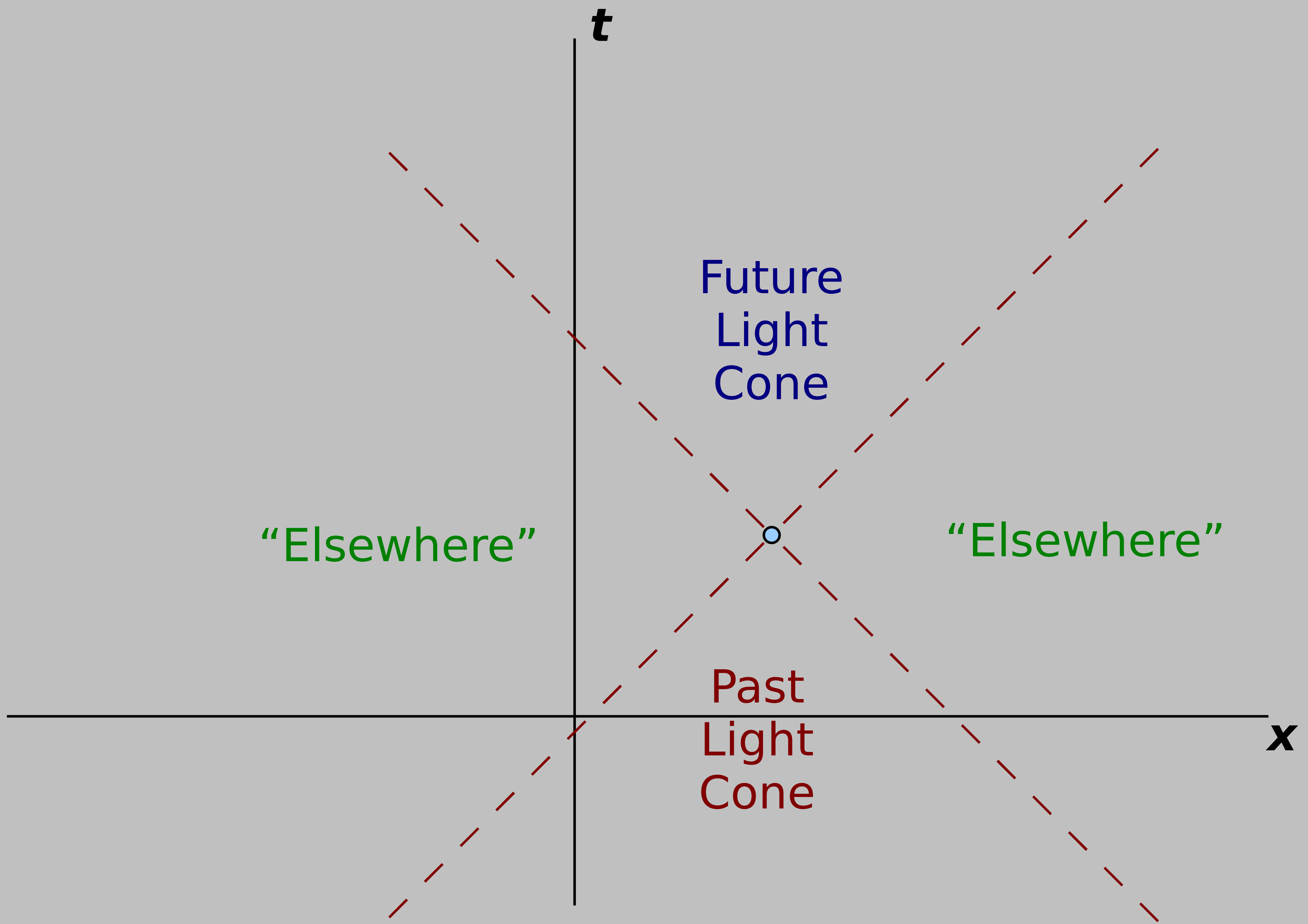
The x and t axes for faster and faster observers “close up” around a 45° angle.

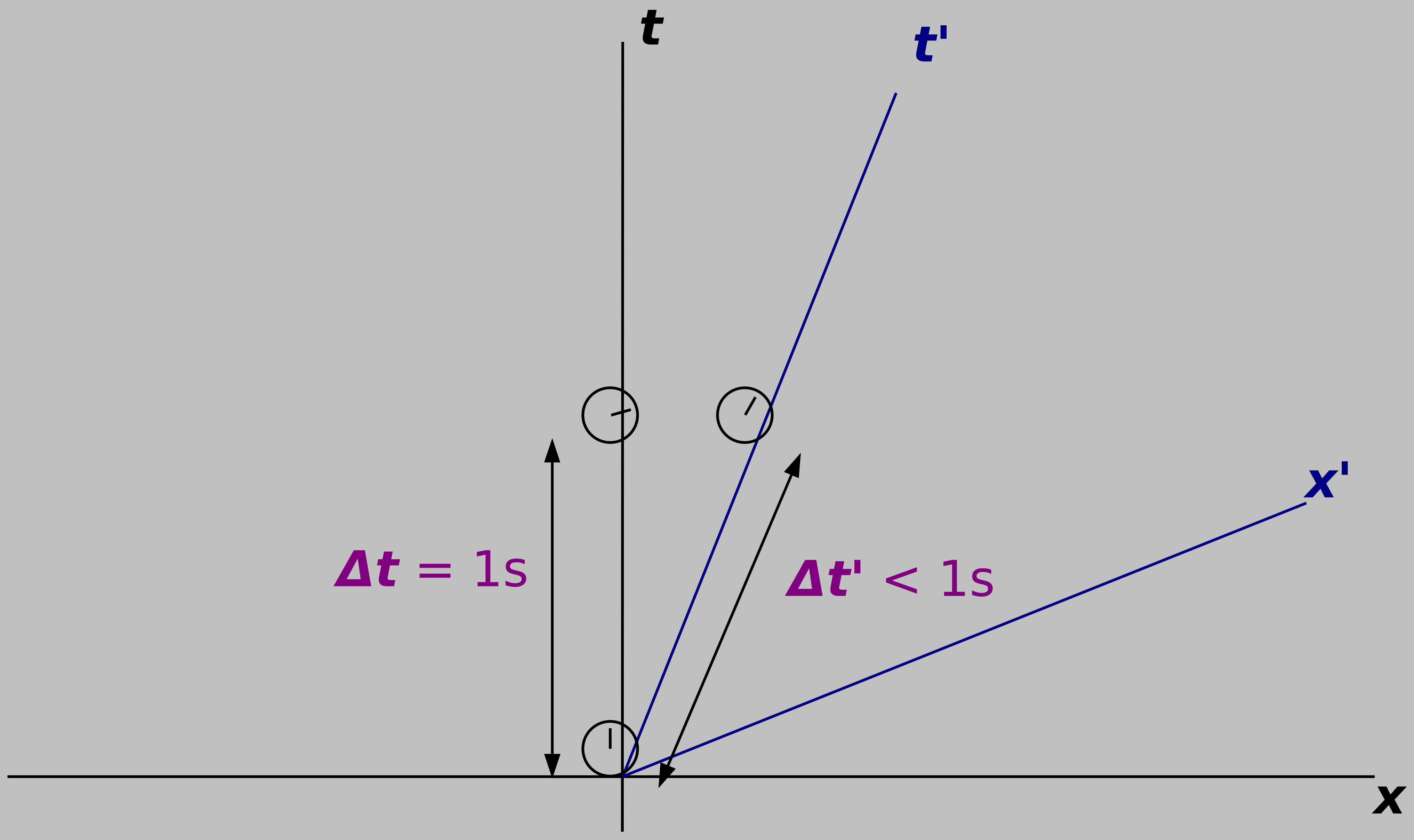


“Lines of simultaneity” in the *primed* frame are parallel to the ***x'*** axis.


t
Light always moves at c, so the world line for light is always at a 45° angle.
x

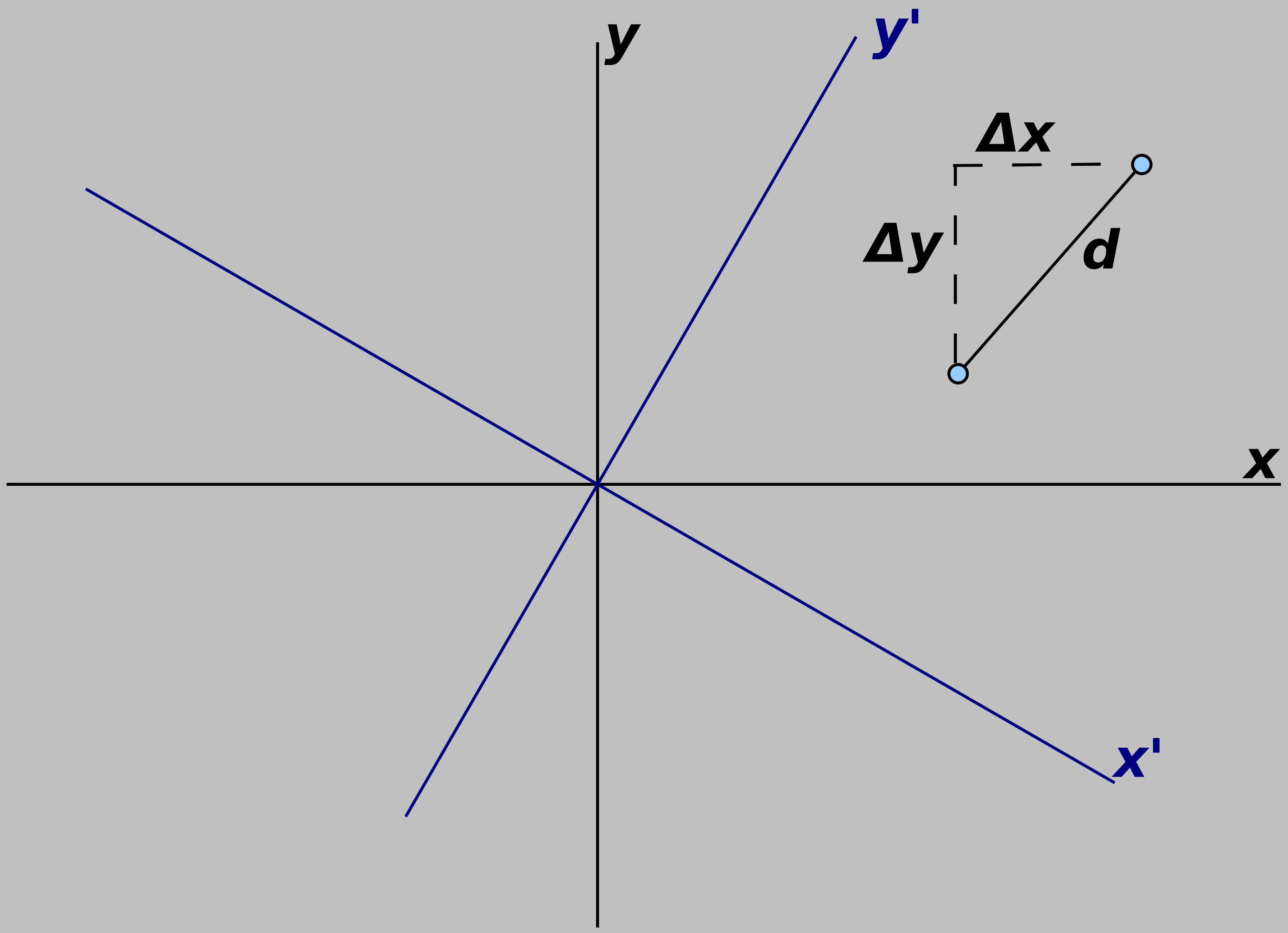
Light Cones for an Event



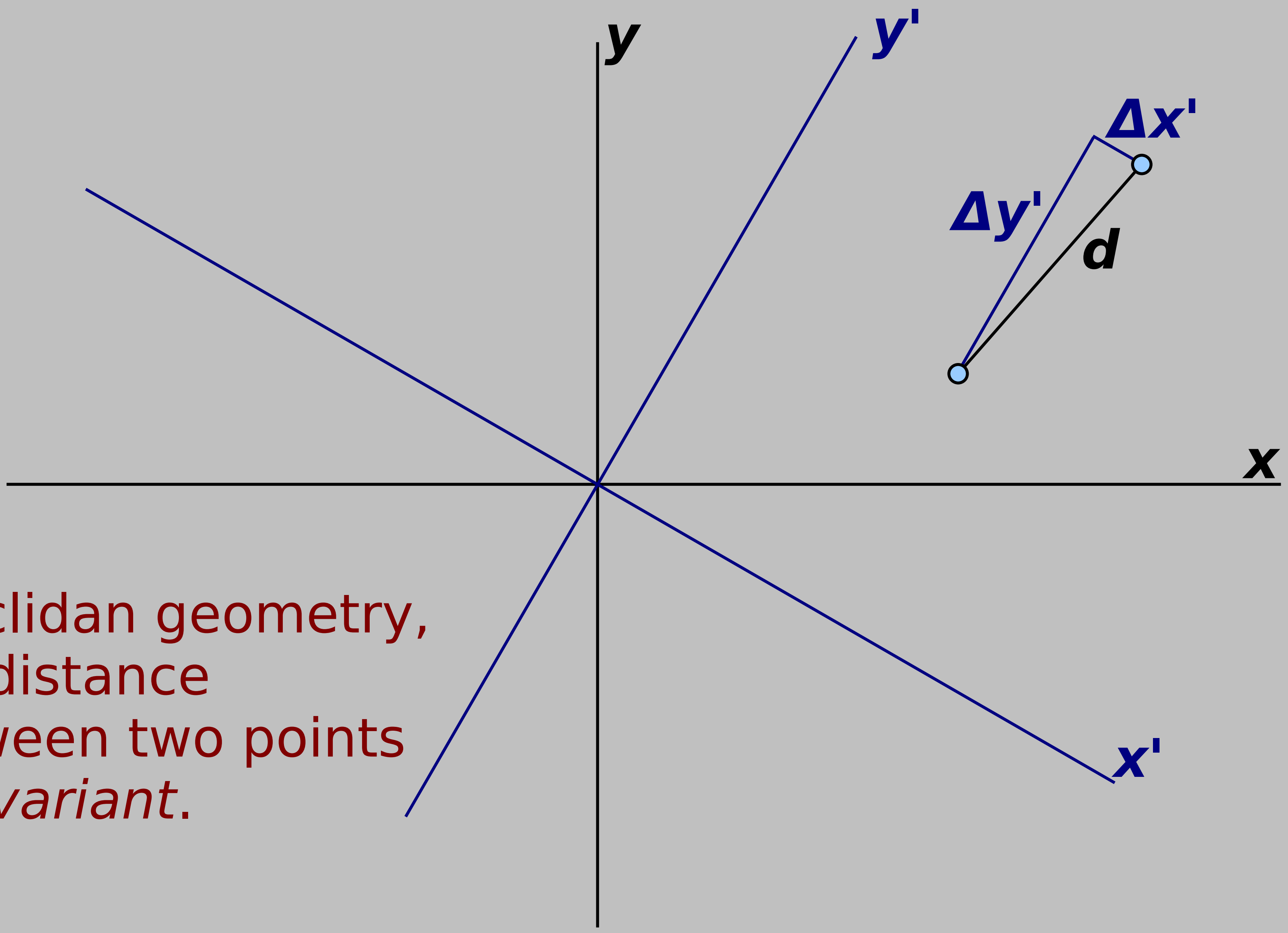


Moving clocks run slow – this implies that the axis scaling in the primed frame is not obvious!

Euclidean Geometry – Space Space diagrams

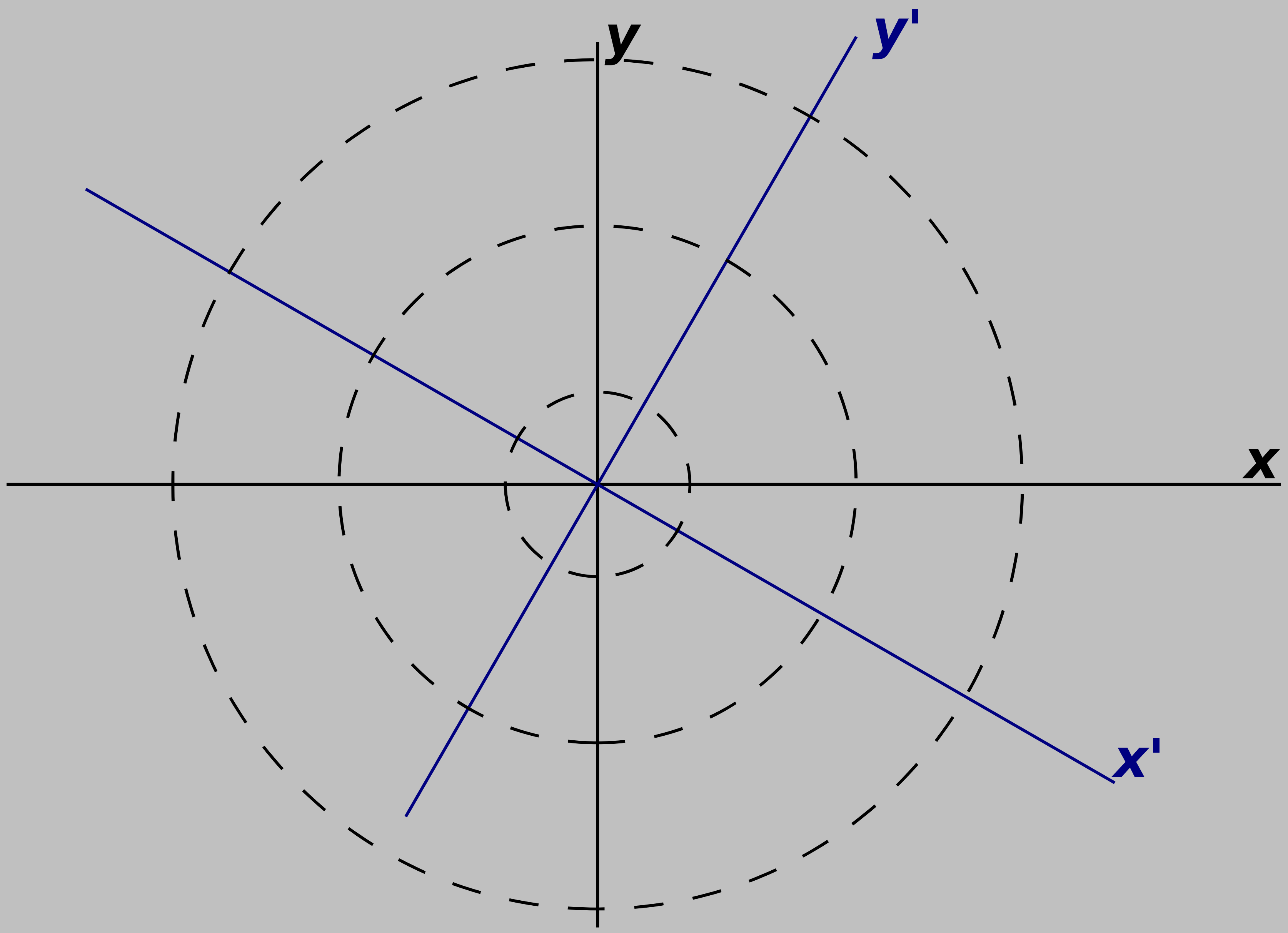


$$d^2 = \Delta x^2 + \Delta y^2 \qquad d = \sqrt{\Delta x^2 + \Delta y^2}$$



In Euclidan geometry, the distance between two points is *invariant*.

$$d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(\Delta x')^2 + (\Delta y')^2}$$



All points the same invariant distance from the origin form a circle.

$$r^2 = x^2 + y^2 = (x')^2 + (y')^2$$

Special Relativity – the "invariant interval"

In relativity, we have a more complicated invariant...

$$(\Delta s)^2 = (\Delta x)^2 - (c\,\Delta t)^2$$

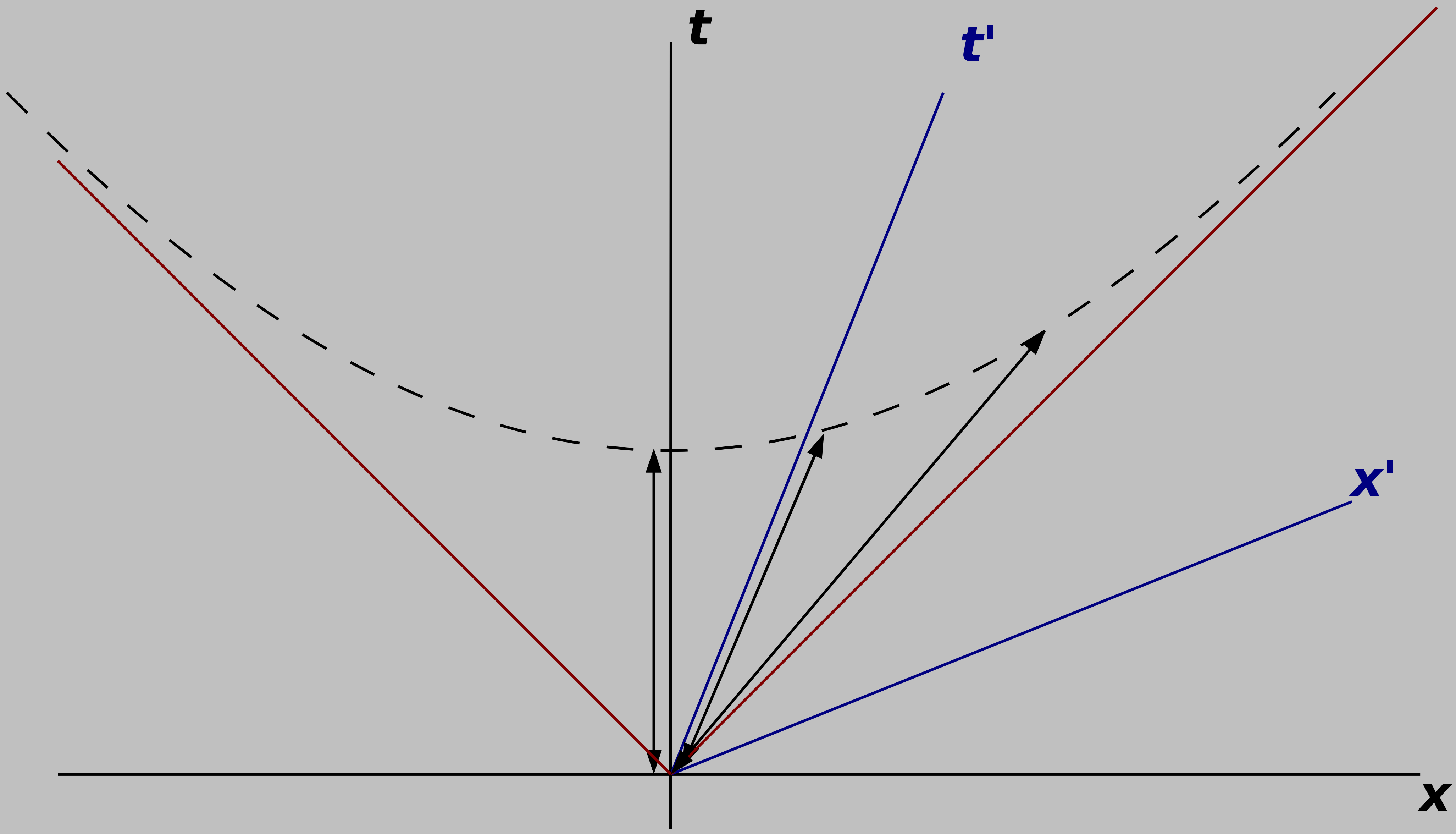
Δx and ***Δt*** are frame-depending, but everybody will measure exactly the same ***Δs***.

$$(\Delta s)^2 = (\Delta x)^2 - (c\,\Delta t)^2 = (\Delta x')^2 - (c\,\Delta t')^2$$

Or, if you consider all three dimensions...

$$\Delta s^2 = -c^2 \Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$$

“Invariant Hyperbolae”



SUMMARY

- Spacetime diagrams are a powerful tool for visualizing the results of relativity
- The vertical axis is time, the horizontal axis is space
- The ***t***-axis is the "world line" of the unprimed observer
- Something moving at constant velocity– say the ***t'*** axis, or the world line of the primed observer– is a sloped straight line
- Light is a 45° line
- Moving observers have x and t axes that "close in" on the light-line
- The invariant interval

$$\Delta s^2 = -c^2 \Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$$